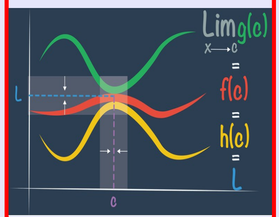
Math 261

Spring 2023

Lecture 28



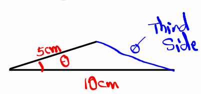
Feb 19-8:47 AM

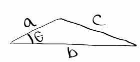
In a triangle ABC, Two sides are 5 cm and 10 cm.

The angle between them changes at 1° per min. $\frac{d\theta}{dt} = 1°/\text{min.} = \frac{\pi}{180}/\text{min.}$

How fast is the third side changing when that angle is 30°?

Law of Cosines
SAS





$c^2 = a^2 + b^2 - 2ab\cos\theta$

$c^2 = 5^2 + 10^2 - 2\cdot 5\cdot 10\cdot\cos\theta$

$c^2 = 25 + 100 - 100\cos\theta$

$c^2 = 125 - 100\cos\theta$

$c^2 = 125 - 100\cdot\cos 30°$
$= 125 - 100\cdot\frac{\sqrt{3}}{2}$
$= 125 - 50\sqrt{3}$

$\boxed{c^2 \approx 38.4}$

$\boxed{c \approx 6.1}$

$\frac{d}{dt}[c^2] = \frac{d}{dt}[125] - 100\frac{d}{dt}[\cos\theta]$

$2c\frac{dc}{dt} = 0 - 100\cdot(-\sin\theta)\cdot\frac{d\theta}{dt}$

$2c\frac{dc}{dt} = 100\cdot\sin\theta\cdot\frac{d\theta}{dt}$

for $\theta = 30°$ $\quad 2\cdot 6.1\cdot\frac{dc}{dt} = 100\cdot\sin 30°\cdot\frac{\pi}{180}$

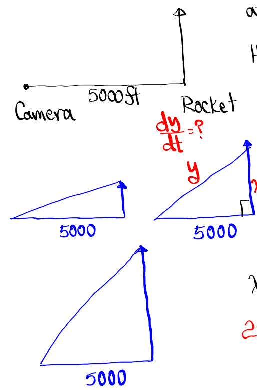
$12.2\frac{dc}{dt} = \cancel{100}^{\cancel{50}\,5}\cdot\frac{1}{\cancel{2}}\cdot\frac{\pi}{\cancel{180}_{18}}$

Related Rates

$\frac{dc}{dt} = \frac{\frac{5\pi}{18}}{12.2} = \frac{.873}{12.2} \approx .072$ cm/min.

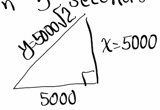
Mar 27-9:40 AM

A Camera is on the ground 5000 ft away from the launching Pad for rocket that goes up vertically. Rocket is rising vertically at 1000 ft/sec. How fast is the distance between the Camera and the rocket changing in 5 seconds?



$\frac{dx}{dt} = 1000$ ft/Sec.

in 5 Seconds



$$x^2 + 5000^2 = y^2$$

$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$x\frac{dx}{dt} = y\frac{dy}{dt}$$

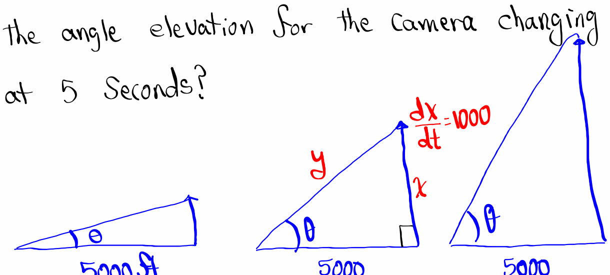
$$5000 \cdot 1000 = 5000\sqrt{2}\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{\cancel{5000} \cdot 1000}{\cancel{5000}\sqrt{2}}$$

$$= \boxed{500\sqrt{2} \text{ ft/Sec.}}$$

Mar 28-8:48 AM

What if we want to know how fast is the angle elevation for the Camera changing at 5 Seconds?



$$\left.\frac{d\theta}{dt}\right|_{t=5}$$

$$\tan\theta = \frac{x}{5000} \qquad 5000\tan\theta = x$$

$$5000\frac{d}{dt}[\tan\theta] = \frac{dx}{dt}$$

$$5000 \cdot \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$$

when $t = 5 \Rightarrow x = 5000$

$$\tan\theta = \frac{5000}{5000}$$

$$\theta = 45^\circ$$

$$5000 \cdot \sec^2 45^\circ \cdot \frac{d\theta}{dt} = 1000$$

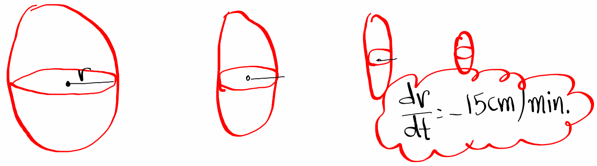
$$5000 \cdot (\sqrt{2})^2 \frac{d\theta}{dt} = 1000$$

$$\frac{d\theta}{dt} = \frac{1000}{10000}$$

$$\boxed{\frac{d\theta}{dt} = \frac{1}{10} \text{ Rad/Sec.}}$$

Mar 28-8:57 AM

A spherical balloon is to be deflated so that its radius decreases at constant rate of 15 cm/min.



At what rate is air being removed when $\boxed{\text{radius is } 9 \text{ cm}}$?

Volume

$V = \frac{4}{3}\pi r^3$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot \frac{d}{dt}[r^3]$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi \cdot 9^2 \cdot (-15)$$

$$\boxed{\frac{dV}{dt} = -4860\pi \ \text{cm}^3/\text{min.}}$$

Mar 28-9:06 AM

Point P is moving along the path given by the curve $y = \sqrt{x^3 + 17}$. $\frac{dy}{dt} = 2$

$\boxed{y \text{ is increasing at the rate of } 2 \text{ units/sec}}$

At (2,5), how fast x is changing?

Is x increasing or decreasing?

$$y = \sqrt{x^3 + 17} \rightarrow y^2 = x^3 + 17$$

$$\frac{d}{dt}[y^2] = \frac{d}{dt}[x^3 + 17]$$

$$2y \frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$

at (2,5)

$$2 \cdot 5 \cdot 2 = 3(2)^2 \cdot \frac{dx}{dt}$$

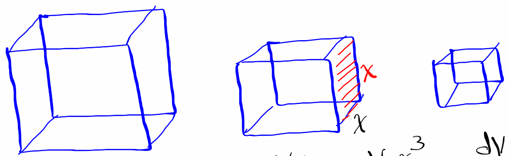
$$\frac{dx}{dt} = \frac{5}{3} \text{ units/sec.}$$

Increasing

$$\frac{dx}{dt} > 0$$

Mar 28-9:14 AM

At a certain instant, the edge of a cube is 5 inches and its volume is decreasing at 2 in^3/min.



Volume $V = x^3$

$\frac{dV}{dt} = -2\ in^3/min$ when $x = 5$ in.

How fast is its surface area changing? Increasing or Decreasing?

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$-2 = 3 \cdot 5^2 \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{-2}{75}$

$S = 6x^2$

$\frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$

$= 12(5) \cdot \frac{-2}{75}$

$\boxed{\frac{dS}{dt} = -1.6\ in^2/min.}$

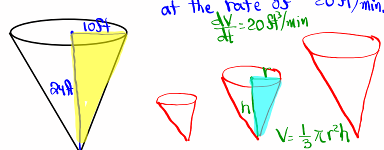
Decreasing $\frac{dS}{dt} < 0$

Mar 28-9:20 AM

A conical water tank with vertex down has a radius of 10 ft at the top, and 24 ft high.

water pours into the tank at the rate of 20 ft^3/min.

$\frac{dV}{dt} = 20\ ft^3/min$



$V = \frac{1}{3}\pi r^2 h$

$\frac{10}{24} = \frac{r}{h}$

$24r = 10h$

$r = \frac{10h}{24}$

$r = \frac{5}{12}h$

How fast is the depth of water increasing when the water is 16 ft deep?

$\frac{dh}{dt} = ?$ when $h = 16$

$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$

$V = \frac{25\pi}{432} h^3$

$\frac{dV}{dt} = \frac{25\pi}{432} \cdot 3h^2 \frac{dh}{dt}$

$20 = \frac{25\pi}{\cancel{432}_{144}} \cdot \cancel{3} \cdot 16^2 \frac{dh}{dt}$

$20 = \frac{6400\pi}{144} \frac{dh}{dt}$

$2880 = 6400\pi \frac{dh}{dt}$

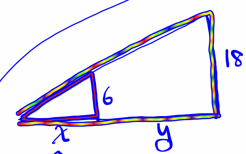
$\frac{dh}{dt} = \frac{\cancel{2880}}{\cancel{6400}\pi}$ (144, 72, 18, 9 / 320, 160, 40, 20)

$\frac{dh}{dt} = \frac{9}{20\pi}$ ft/min.

Mar 28-9:29 AM

A man 6 ft tall is walking at the rate of 3 ft/sec $\boxed{\text{towards}}$ a streetlight 18 ft high.

How fast is the length of the shadow changing?



$\frac{dx}{dt} = ?$ $\qquad \frac{dy}{dt} = -3 \text{ ft/sec.}$

$$\frac{x}{\cancel{6}_{1}} = \frac{x+y}{\cancel{18}_{3}}$$

$$3x = x + y$$

$$2x = y$$

$$2 \cdot \frac{dx}{dt} = \frac{dy}{dt}$$

$$2 \cdot \frac{dx}{dt} = -3$$

$\frac{dx}{dt} = -1.5$ ft/sec.

Length of shadow is decreasing

Mar 28-9:45 AM